LXXXI. *Some Confiderations on a late Treatife intituled,* A new Set of Logarithmic Solar Tables, *&c. intended for a more commodious Method of finding the Latitude at Sea, by Two Obfervations of the Sun* ; *by* H. Pemberton, *M. D. R. S.* Lond. *et R. A.* Berol. *S.*

Read Nov. 20, 1760.

AS it happens not unfrequently at fea for the unfeafonable intervention of clouds to prevent the ordinary method of determining the fhip's latitude by the fun's meridian altitude, even when it is of primary confequence, that the true latitude fhould be known ; a problem for remedying this difappointment is ftated in many treatifes of navigation, for finding the latitude of a place by any two altitudes of the fun, with the interval of time between them.

A problem fimilar to this is propofed, and folved inftrumentally upon a globe, by a very early writer, Petrus Nonius (*a*), namely, to find the latitude by two altitudes of the fun, and the angle made by the azimuth circles paffing through the fun, when the altitudes are taken. And fince more commodious and accurate inftruments for meafuring time have been invented, than were known to this author, the other problem has been propofed for the fame pur-

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(*a*) In Libr, De Obferv, Regul. et Inftru n. Geometr. lib. ii. c. 14.

pofe,

pofe, of which a conftruction upon the principles of
the ftereographic projection of the fphere is exhibited
by Mr. Collins, in his Mariner's Plain Scale new
planed (*b*). And as the direct method of folving both
thefe problems by numbers requires a diverfity of tri-
gonometrical operations, a fet of tables has lately been
publifhed for a more compendious way of computa-
tion in the problem, where the interval of time is
given, whereby the fhip's true latitude may be very
expeditioufly derived from the fhip's dead reckoning,
provided the obfervations are made within certain li-
mits of time.

But however worthy of notice this method may
be, new tables for the purpofe are altogether unne-
ceffary. It confifts of two parts: the firft computes,
from the latitude exhibited by the dead reckoning of
the fhip, the diftance from noon of the middle time
between the obfervations, and thence the time of
either: the fecond operation computes, from one of
thefe obfervations, what fhould be the fun's meri-
dian altitude, had the fhip's reckoning given the true
latitude; but if the latitude affumed from that reckon-
ing is erroneous, the altitude thus computed will not
be conformable to it; however, if the times for the
obfervations are properly chofen, it will much better
agree to the true latitude, and thereby the affumed
latitude may be more or lefs corrected.

But both thefe operations are an immediate con-
fequence from the propofition in fpherical trigono-
metry, ufually delivered under the name of the fourth
axiom, which is this; That the fquare of the radius
is to the rectangle under the fines of the fides con-

(*b*) Book iii. p. 35.

talning

taining any angle, as the versed sine of that angle to the difference between the versed sines of the third side, and of the difference between the sides containtaining the angle.
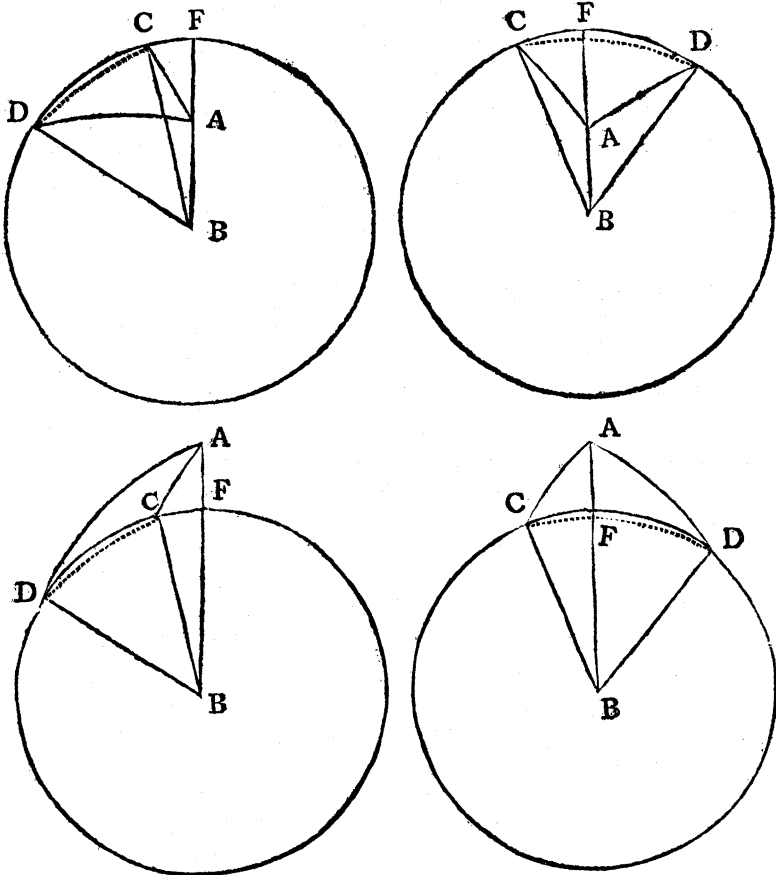
Hence if A be the zenith of any place; B the elevated pole; C, D two places of the sun in the parallel of declination D F; by this proposition in the triangle A B C,

3   Rad.

Rad.^{q} : fin. A B × fin. B C (cof. lat. × cof. decl. ☉)
:: verf. f. A B C : verf. f. A C — verf. f. A F;

And, in like manner, in the triangle A B D,

Rad.^{q} : cof. lat. × cof. decl. ☉ :: verf. f. A B D
: verf. f. A D — verf. f. A F;

Confequently,

Rad.^{q}: cof.lat.×cof.decl.☉ : :verf.f. ABD — verf.f. ABC
: verf. f. A D — verf. f. A C.

But $\frac{1}{2}$ rad. × $\overline{\text{verf. f. A B D — verf. f. A B C}}$ is =
fin. $\dfrac{\text{A B D} + \text{A B C}}{2}$ × fin. $\dfrac{\text{A B D} - \text{A B C}}{2}$;

Therefore,

Rad.^{q} : cof. lat. × cof. decl. ☉ :: fin. $\dfrac{\text{A B D} + \text{A B C}}{2}$

× fin. $\dfrac{\text{ABD} - \text{ABC}}{2}$: $\frac{1}{2}$ rad. × $\overline{\text{verf.f. AD} - \text{verf.f. AC.}}$

And this is the firft operation in the treatife, thefe re-
marks concern. For in the rectangle, the fin. $\dfrac{\text{ABD} + \text{ABC}}{2}$

× the fin. $\dfrac{\text{A B D} - \text{A B C}}{2}$, the firft fide, when the obfer-
vations are both made on the fame fide of noon, and
the fecond fide of this rectangle, when one obferva-
tion is before noon, and the other after, is the fine
of the diftance of the middle time between the ob-
fervations from noon, and the other fide is the fine
of half the diftance between the obfervations. And
in the new tables one column exhibits the arithme-
tical complement of the logarithmic fine of half the
elapfed time, and the next, the logarithm of twice
the fine of the middle time.

But moreover, $\frac{1}{2}$ rad. × $\overline{\text{verf. f. A D — verf. f. A C}}$
being equal to fin. $\dfrac{\text{A D} + \text{A C}}{2}$ × fin. $\dfrac{\text{A D} - \text{A C}}{2}$, the
operation may be expreffed more commodioufly thus,
Rad.

Rad.^{q} : cof. lat. × cof. decl. ⊙ : : fin. $\dfrac{ABD + ABC}{2}$

× fin. $\dfrac{ABD - ABC}{2}$: fin. $\dfrac{AD + AC}{2}$ × fin. $\dfrac{AD - AC}{2}$;

now requiring the common logarithmic fines only.

Again, the fquare of the radius being, as above, to cof. lat. × cof. decl. ⊙, as the verf. f. A B C to verf. f. A C — verf. f. A F, or as verf. f. A B D to verf. f. A D — verf. f. A F, thefe analogies exprefs the fecond operation, which, in this treatife, is unneceffarily confined to the leffer angle A B C. The column in the table intituled Rifing confifts of logarithmic verfed fines, which may be applied to either of the angles A B C or A B D promifcuoufly; for here A F being equal to the excefs of B C or B D above the arch B A, affumed for the complement of the latitude, the arches A C and A D, in the preceding analogies, will be the complements of the altitudes obferved, if the latitude were truly affumed, otherwife not; but the difference of their verfed fines, will however be equal to the difference of the verfed fines of the complements of the true latitudes; for this is fuppofed in the firft operation. Therefore, if one of the fourth terms of thefe analogies be deducted from the verfed fine of the complement of the greater altitude, or the other from the verfed fine of the complement of the leffer, the remainder will be the fame, and exhibit a verfed fine for the complement of the fun's altitude different from A F, when B A is affumed different from the true latitude, and nearer to the truth, if the times for the obfervations are properly chofen.

But farther, the two preceding analogies may be reduced to thefe;

Rad.

Rad.^{q} : cof. lat. × cof. decl. ⊙ :: $\overline{\text{fin. } \frac{1}{2}\text{A B C}}$ ^{q}
: $\frac{1}{2}$ rad. × $\overline{\text{verf. f. A C}}$ — verf. f. A F,
And

Rad.^{q} : cof. lat. × cof. decl. ⊙ :: $\overline{\text{fin. } \frac{1}{2}\text{ A B D}}$ ^{q}
: $\frac{1}{2}$ rad. × $\overline{\text{verf. f. A C}}$ — verf. f. A F ;
Alfo,

Verf. f. A C — verf. f. A F = cof. A F — cof. A C, and
verf. f. A D — verf. f. A F = cof. A F — cof. A D.

And thus the common tables of fines will fupply the ufe of this column of the new tables, as well as the preceding.

The firft example of this treatife, wrought by the common tables.

Here the declination of the fun is ftated at 11° 17′ N. the altitude firft taken, being the leffer, 46° 55′, the greater altitude 54° 7′, and the difference of time between the obfervations 1^h 25^m, or 85^m; which, divided by 4, gives 21° 15′ for the arch of the equinoctial correfponding.

Here the obfervations are both before noon; therefore, in fig. 1. the latitude being affumed 46° 50′,

For the firft operation.

$$
\begin{array}{rrrr}
 & \overset{\circ}{} & \overset{\prime}{} & \overset{\prime\prime}{} \\
\text{A C} = & 35 & 53 & 0 \\
\text{A D} = & 43 & 5 & 0 \\
\text{Their fum} = & 78 & 58 & 0 \\
\text{Their difference} = & 7 & 12 & 0 \\
\end{array}
$$

Sine of $\frac{1}{2}$ fum	(39	29	0)	9.80336
Sine of $\frac{1}{2}$ difference	(3	36	0)	8.79789
Arith. com. of the fine of $\frac{1}{2}$ C B D (= 10	37	30)		0.73429
Arith. com. of the fine of B F, or the cof. of the decl. of the fun, - - - - - - - - - - - - - - $\Big\}$				0.00848
Sum				19.34402
Cof. of the lat. affumed				9.83513
Diff. = fin. $\dfrac{\text{A B D} + \text{A B C}}{2}$				9.50889

For

For the second operation.

$$\frac{A B D + A B C}{2} = \begin{array}{ccc} ° & \prime & \prime\prime \\ 18 & 49 & 50 \end{array}$$

$$\frac{A B D - A B C}{2} = 10 \ 37 \ 30$$

$$A B C = 8 \ 12 \ 20$$

Sine of $\frac{1}{2}$ A B C (4 6 10) $\left\{ \begin{array}{l} 8.85458 \\ 8.85458 \end{array} \right.$

Cof. of decl. ☉   9.99152

Cof. of the lat. affumed   9 83513

Sum – 30.00000 = log. of the $\dfrac{\text{verf.f. AC} - \text{verf.f. AF}}{2}$ 7.535⁸1

Verf. f. A C — verf. f. A F	0.006868
Natural cof. of A C, the complement of the greater altitude, - - - - - - - - - - - - -	0.810212
Natural cof. of 35° 12′ 23″	0.817080

Or, $\dfrac{A B D + A B C}{2} + \dfrac{A B D - A B C}{2}$ being $=$ A B D,

Sine of $\frac{1}{2}$ A B D (14° 43′ 40″) $\left\{ \begin{array}{l} 9.40522 \\ 9.40522 \end{array} \right.$

Cof. of the decl. of the fun   9 99152

Cof. of the latitude affumed   9.83513

Sum – 30.00000 = log. of the $\dfrac{\text{verf f. AD} - \text{verf f. AF}}{2}$ 8.63709

Verf. f. A D — verf. f. A F	0 086720
Natural cof. of A D, the complement of the leffer altitude, - - - - - - - - - - - - -	0.730361
Natural cof. of 35° 12′ 23″, as before	0.817081

Thus the meridian diftance of the fun from the zenith is computed at 35° 12′ 23″, and this added to the fun's declination 11° 17′ N. gives 46° 29′ 23″ for the latitude, different from that affumed, and nearer to the true latitude; which, by a fecond operation

4

operation with this latitude now found, will approach ftill nearer to the truth, thus:

The fum of the four firft logarithms in the firft of the two former operations, - - - - - - - } 19.34402

$$\text{Cofine of } 46°\ 29'\ 23'' \qquad 9.83789$$

$$\text{Sine of } \frac{ABD + ABC}{2} = 18\ 42\ 25 \qquad 9.50613$$

$$\frac{ABD - ABC}{2} = 10\ 37\ 30$$

$$ABC = 8\ \ 4\ 55$$

$$\tfrac{1}{2}ABC = 4\ \ 2\ 27\tfrac{1}{2} \left\{ \begin{array}{c} 8.84800 \\ 8.84800 \\ 9.99152 \\ 9.83789 \end{array} \right.$$

$$\text{Log. } \frac{\text{verf. f. } AC - \text{verf. f. } AF}{2} \qquad 7.52541$$

Verf. f. AC — verf. f. AF	0.006704
Natural cof. of AC, as before,	0.810212
Natural cof. of $35°\ 13'\ 21''$	0.816916

The declination of the fun added to this, amounts to 46° 30' 31", the true latitude, as directly computed, being 46° 30' 19".

However, it muft not be expected, that this method of computation will always converge thus expeditiously to the true latitude. Had thefe obfervations been made about 3 hours fooner in the day, and with the fame interval between them, if the firft altitude of the fun had been found 18° 55', and the fecond 33° 11', the latitude computed would come out wider from the truth, than that affumed, one exceeding, and the other falling fhort of the true latitude, which will lie between them, and is nearly the fame, as above. But if the leffer altitude had

been

been 29° 13′, and the greater 40°; in 9 degrees of north latitude the interval between the obfervations would have been nearly the fame, as before. But if this latitude be fought by any latitude affumed near it, the latitude computed according to the method above, will fall more remote from the truth, than that affumed, and err the fame way.

In general, the error in the latitude affumed will bear to the error in the latitude computed, nearly the ratio compounded of that of the rectangle under the radius and the cofine of the diftance from noon of the middle point of time between the two obfervations to twice fin. ½ A B C × fin. ½ A B D, and the ratio of rad. × fin. A F to fin. B C × cof. A B; infomuch that the diftance of the middle point of time between the two obfervations from noon is to be confidered, as the limit, where thefe computations fhall ceafe to converge, when the rectangle under the radius, and the cofine of this diftance from noon fhall be to twice fin. ½ A B C × fin. ½ A B D, (or the cofine of this diftance to the difference between the verfed fine of this diftance from the verfed fine of half CBD) as fin. BF × cof. AB to rad. × fin. A F. And the errors in the affumed and computed latitudes fall on different fides of the true latitude, when both the obfervations are on the fame fide of noon, and the zenith lies between the meridian fun and the elevated pole; or when the obfervations are one before, and the other after noon, if the meridian fun paffes between the zenith and the elevated pole: otherwife, they fall on the fame fide of the true latitude.

However,

However, the direct method of computation, by the affiftance of the natural fines, will not be fo much more operofe than this compendium, as may at firft fight be imagined. For the arch of a great circle being drawn through C and D, forming the triangle B C D, if the logarithmic cofine of B C or B D be added to the logarithmic tangent of half the angle C B D, and the logarithmic fine of B C or B D be added to the logarithmic fine of half this angle, the firft fum is the logarithmic cotangent of the angle B C D, and the fecond the logarithmic fine of half C D, the bafe of the triangle. Then, in the triangle A C D, from the fides, now all given, is to be computed the angle A C D, the difference between which, and the angle above found B C D, is the angle B C A, when the zenith lies between the pole, and the great circle through C, D; but when the zenith lies beyond that circle, the angle B C A is either the fum of thofe angles, or the fupplement of that fum to a circle. And, in the laft place, if twice the logarithmic fine of half this angle, and the logarithmic fines of B C and A C are added together, the fum, after thrice the logarithm of the radius has been deducted, is the logarithm of half the excefs of the natural cofine of B C ∽ A C above the natural cofine of A B, or the natural fine of the latitude, according to the trigonometrical axiom, which has been above referred to; for rad. × ½ verf. ∠ B C A is equal to $\overline{\text{fin. } \frac{1}{2} \text{ B C A}}|^q$ (c).

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(c) Vide Neper. Mirif. Canon. Conftruct. Edinburg. 1619, p. 59.

The

The operation for the preceding example will be this:

Tang. ½ C B D —	9.273225		Sine ½ C B D —	9.265714
Cof. B C —	9.291504		Sine B C —	9.991524
Cotang. B C D —	9.564729		Sine ½ C D —	9.257238
B C D = 87° 53′ 52″			½ C D = 10° 25′ 2½″	

$$
\begin{array}{rrrll}
 & \overset{\circ}{} & \overset{\prime}{} & \overset{\prime\prime}{} & \\
A D = & 43 & 5 & 0 & \\
A C = & 35 & 53 & 0 & - \quad 0.232001 \\
C D = & 20 & 50 & 5 & - \quad 0.448950 \\
\text{Sum} = & 99 & 48 & 5 & \\
\tfrac{1}{2}\,\text{Sum} = & 49 & 54 & 2\tfrac{1}{2} & - \quad 9.883621 \\
\tfrac{1}{2}\,\text{Sum} - A D = & 6 & 49 & 2\tfrac{1}{2} & - \quad 9.074466 \\
 & & \text{Sum} & - & 19.639038 \\
 & & \tfrac{1}{2}\,\text{Sum} & - & 9.819519
\end{array}
$$

ar.co. of the fines. { 0.232001, 0.448950 }

fines. { 9.883621, 9.074466 }

This half fum is the logarithmic cofine of half A C D; therefore, half A C D is = 48° 42′ 11″, and its difference from ½ B C D will give ½ A C B = 4° 45′ 15″.

$$
\begin{array}{ll}
\text{Sine of } \tfrac{1}{2}\,A C B \left\{ \begin{array}{l} 8.918444 \\ 8.918444 \end{array} \right. \\
\text{Sine of A C} - & 9.767999 \\
\text{Sine of B C} - & 9.991524 \\
\hline
0.0039483 - \text{log.} - & 7.596411 \\
\quad 2
\end{array}
$$

0.0078966

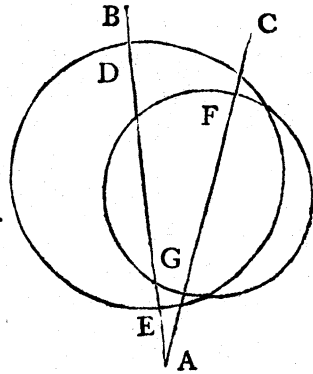0.7333345 — natural cofine of B C — A C, that is, of 42° 50′.

0.7254379 — natural cofine of A B, or fine of the latitude.

Therefore, the latitude is 46° 30′ 19″.

But farther, as in the cafes, where the above approximation takes place, it is an advantage, that the affumed latitude fhould be taken as near the true one,

one, as can be, the dead reckoning of the ſhip need not be intirely relied on for that purpoſe; for the inſtrumental conſtruction propoſed by Mr. Collins (d) will very readily give the true latitude, as nearly as the inſtruments uſed can expreſs: and his method may be deſcribed thus.

Two ſtraight lines AB, AC being drawn in an angle cor-reſponding with the diſtance in time between the obſerva-tions, and in one of the lines, as A B, the lengths A D, A E being taken from any ſcale of tangents, one equal to the tangent of half the ſum, and the other equal to the tangent of half the difference of the diſtance of the ſun from the pole, and from the zenith of the firſt obſervation, and alſo the points F, G, taken in the other line in the ſame manner related to the other obſervation; then circles being deſcribed on the two diameters D E, F G, the diſtance of A from one of the interſections of theſe circles will be the tan-gent of half the complement of the latitude.

Moreover, as in the treatiſe, which has given oc-caſion to this diſcourſe, it is propoſed ſometimes to take into conſideration the ſhip's motion during the time, between the obſervations, but imperfectly, re-gard being had to the change made by the ſhip in longitude only; but the change in latitude alſo may be taken into conſideration in the foregoing con-

(d) In the place above cited.

ſtruction,

ftruction, by making the angle B A C equal to the sum or difference of the interval of time, and the change in the ship's longitude, according as that change is made towards the west, or towards the east, and then drawing a line from A, not to the interfection of the circles; but so, that the portions of that line, terminated by each of the circles, may be the tangents of arches, whose difference is half the change in the latitude; which, if made from the pole, requires the portion of the line terminated at the circle, whose diameter is D E, to be shorter than the other; and the contrary is required, when the motion of the ship is towards the pole.

For determining the latitude by calculation, if the distance of the sun in the first observation from the zenith of the second could be found, this case would be reduced to the first, wherein the ship is confidered, as stationary. And for this purpose, it has been proposed to make an additional observation, by an azimuth compass, of the angle, the ship's course makes with the azimuth of the sun, when the first altitude is taken; and, perhaps, the same angle may be found with sufficient exactnefs from the latitude in the first obfervation affumed, whether from the dead reckoning, or from the foregoing conftruction.

Why I have taken no notice of the calculations exhibited by the author of the piece here animadverted on, as proofs of his method, will readily appear to those, who shall cast their eye upon them.

S C H O L I U M.

The axiom in trigonometry, on which the calculations here difcourfed of have been shewn imme-
diately

diately to depend, was introduced by Regiomontanus, and is ftill retained, as the foundation of the prefent methods of computing logarithmically an angle from the three fides of a fpherical triangle given, though they may be demonftrated more directly by the following lemma.
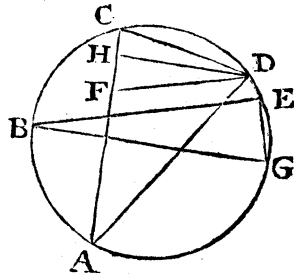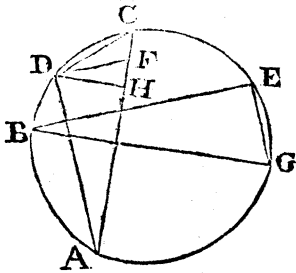
In the circle A B C, the chord A C being drawn, and the arch A B C bifected in B, if D be taken in the circle at pleafure, and the lines B E, D F drawn alfo at pleafure parallel to each other, one terminated by the circle in E, and the other by the chord A C in F; then A D, D C being drawn, B E × D F is = A D × D C.

Draw D H perpendicular to A C, and alfo the diameter B G, joining E G. Then the angles F D H, E B G are equal, and the angles D H F, B E G right. Therefore B E is to B G as D H to D F, and B E × D F = B G × D H = A D × D C.
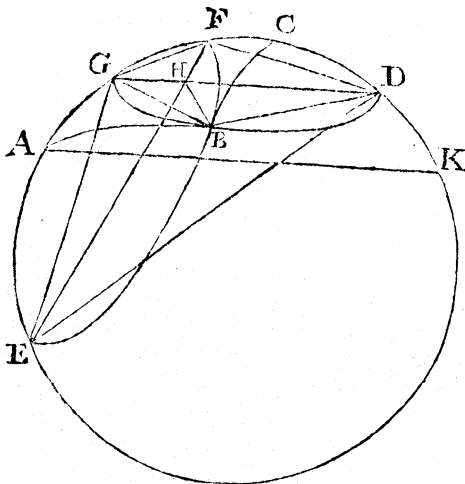
Now, in the spherical triangle A B C, through B, and to the poles A, C, let the femicircles E B F, G B D be defcribed, and the arch A C completed to a circle. Then the femicircles EBF, G B D will be both perpendicular to the plane of this circle, cut it in their diameters E F, G D, and their common interfection B H be alfo perpendicular to it, and confequently perpendicular to G D, the diameter of the femicircle G B D, whereby G B, D B being joined, $GD \times GH$ is equal to GB^q, and $GD \times DH$ equal to BD^q. Again, A K being drawn parallel to G D, the arch A C K is double A C, G C D double C B, E A D the fum of all the three fides of the triangle, $EG = ED - DG (2CB), GF = AF (AE) + CG (CD) - AC = ED - 2 AC$, and $FD = ED - EF (2 AB)$.

Then E G, G F, F D, E D being joined; in the firft place, A K is to D G as $AK \times GH$ to $DG \times GH$, alfo AK to DG as $AK \times DG$ to DG^q, but $AK \times GH = EG \times GF$ by the preceding lemma, and $DG \times GH = GB^q$; whereby $AK \times DG$ is to DG^q as $EG \times GF$ to GB^q. Hence A K being twice the fine of the arch A C, D G twice the fine of the arch G C, equal to B C, E G twice the fine of half the arch E A G, G F

twice

twice the fine of half the arch G F, and G B twice
the fine of half the angle A C B to the radius of the
circle G B D, which is half G D ; fin. A C × fin. B C
is to rad.^{q} as fin. $\overline{\frac{1}{2}ED - BC}$ × fin. $\overline{\frac{1}{2}ED - AC}$
to $\overline{\text{fin. } \frac{1}{2}ACB}|^q$. And this is Napeir's firſt method
of finding an angle from the three fides given (*e*), as it
is uſually delivered.

Again, A K is to D G, or A K × D G to D G^q, as
A K × D H (E D × D F) to D G × D H, or D B^q.
Therefore, D B being twice the fine of half the angle
B C D, or twice the cofine of half the angle A C B to
the radius of the circle G B D, as fin. A C × fin. C B
to rad.^{q} ſo is $\frac{1}{2}$ E D × fin. $\overline{\frac{1}{2}ED - AB}$ to $\overline{\text{cof. } \frac{1}{2}ACB}|^q$.
And this is Napeir's ſecond method (*f*).

And laſtly, the cofine of an arch or angle being
to its fine, as radius to the tangent, fin. $\frac{1}{2}$ E D
× fin. $\overline{\frac{1}{2}ED - AB}$ will be to fin. $\overline{\frac{1}{2}ED - BC}$
× fin. $\overline{\frac{1}{2}ED - AC}$ as rad.^{q} to $\overline{\text{tang. } \frac{1}{2}ACB}|^q$, which
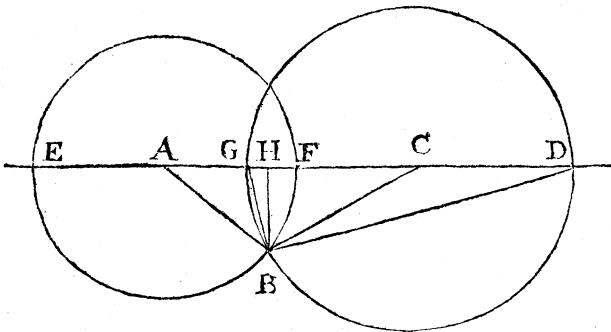is a third method added by Gellibrand (*g*).

Moreover,

In plain triangles, from the three fides given may
an angle be found by a procefs fimilar to each of
thefe, as follows :

(*e*) Mirific. Canon. Log. defcrip. lib. ii. c. 6. § 8.
(*f*) Ibid. § 9.
(*g*) Trigon. Britan. p. 75.

In

In the plain triangle A B C, if on the centers A and C be defcribed circles through B, cutting A C in E, F, G, and D, the perpendicular B H being let fall on A C, E H × H F is equal to G H × H D. Therefore HF : GH :: HD : HE, and by compofition GF : GH :: ED : HE, and :: ED — GF : EG; alfo HF : GF :: HD : ED, and :: DF : ED—GF.

Moreover ED is = AB + AC + CB, EG = ED — 2CB, FD = ED — 2AB, ED — GF = EG + FD = 2ED — 2CB — 2AB = 2AC. Hence GF is to GH as 2AC to EG, whence GF × EG is = 2AC × GH; and DH is to ED as DF to 2AC, that is, 2AC × DH will be = ED × DF.

Thus 2AC : DG (:: 2AC × DG : DG^q) :: 2AC × GH (EG × GF) : DGH (GB^q) and AC × CB : rad.^{q} :: $\overline{\frac{1}{2}ED — CB} × \overline{\frac{1}{2}ED — AC}$: fin. $\frac{1}{2}$ACB$|^q$.

Likewife 2AC : DG (:: 2AC × DG : DG^q) :: 2AC×DH (ED×DF) : GDH (DB^q); therefore AC × CB : rad.^{q} :: $\frac{1}{2}$ED×$\overline{\frac{1}{2}ED—AB}$: cof. $\frac{1}{2}$ACB$|^q$.

And in the laft place $\frac{1}{2}$ED × $\overline{\frac{1}{2}ED — AB}$: $\overline{\frac{1}{2}ED—CB} × \overline{\frac{1}{2}ED—AC}$ :: rad.^{q} : tang. $\frac{1}{4}$ACB$|^q$.

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This laſt propoſition was firſt delivered by Gelli-brand, and the other two not long after by Oughtred, in their reſpective treatiſes of Trigonometry.

But it may farther here be noted, that from two fides of a plain triangle, as A C, C B, with the angle between them A C B, the third fide A B may be found either by the firſt or fecond of the foregoing proportions; for by the firſt may be found E G × G F, that is, A F^q — A G^q, and by the fecond E D × D F, or A D^q — A F^q; whence by a table of ſquares A F ($=$ A B) may be readily found, for A G is equal to A C — B C, and A D $=$ A C $+$ C B.

In like manner in the ſphe-rical triangle from the ſides A C, C B, and the angle A C B, may the third fide be expedi-tiouſly found with the aid of a table of natural ſines, by the axiom referred to in the be-ginning of this diſcourſe, as modelled for this pur-poſe by Napeir, in the propoſition, which has been already referred to (*h*), whereby the axiom is re-duced to this analogy,

Rad.^{q} : fin. A C × fin. C B :: $\overline{\text{fin. } \frac{1}{2}\text{ A C B}}\rvert^q$
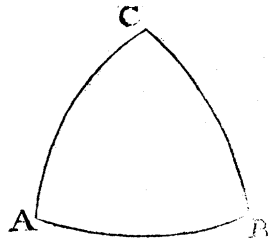
: rad. × $\dfrac{\text{verf. f. A B} - \text{verf. f. } \overline{\text{A C} \backsim \text{C B}}}{2}$ (*i*).

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(*h*) Mirif. Canon. Conſtr. p. 59.

(*i*) An example of this method has been given above in the tri-angle A B C in p. 920. And in default of a table of natural ſines, the baſe A B might have been found thus. The ſum of twice the logarithmic ſine of $\frac{1}{2}$ A C B, added to the logarithmic ſines of A C

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MAY it here be farther added, that thus all the cafes of oblique fpherical triangles may be folved without dividing them into rectangular? For when two fides and the angle included, or when two angles and the fide between them are given, the two other angles or fides may be found by the proportions of Napeir and Briggs, whereby the other two angles or fides are found together (k); the proportions, when two fides with the including angle are given, being thefe,

$$\text{Cof.} \; \frac{AC + CB}{2} : \text{cof.} \; \frac{AC \backsim CB}{2} :: \text{cotang.} \; \tfrac{1}{2}ACB$$
$$: \text{tang.} \; \frac{CAB + CBA}{2}.$$

And

$$\text{Sin.} \; \frac{AC + CB}{2} : \text{fin.} \; \frac{AC \backsim CB}{2} :: \text{cotang.} \; \tfrac{1}{2}ACB$$
$$: \text{tang.} \; \frac{CAB \backsim CBA}{2}.$$

The fecond of thefe has been demonftrated with great concifenefs by Dr. Halley, from the principles of the ftereographic projection of the fphere (l); and the firft is derived by him from the fecond, but not

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and B C amounted to 37.596411. The logarithmic fine of $\frac{AC \backsim BC}{2}$ (9.562468) deducted from half this fum (18.798211) leaves 9.235737. This number fought among the logarithmic tangents exhibits for its correfpondent fine 9.229400, which being deducted from the forefaid half fum (18.79211) leaves 9.568805, which is the logarithmic fine of 21° 44′ 50½″, equal to half A B.

(k) Vide Neper. Mirif. Canon. Conftr. pag. penult.
(l) See Jones's Synopfis, p. 281.

altogether

altogether with the fame fuccefs, by his not obferving, that if the bafe and one of the fides be completed to femicircles, the fecond of the two proportions in the fupplemental triangle thence formed leads directly to the firft in the original triangle.

And to conclude, if two fides, and the angle oppofite to one of them, or two angles with the fide oppofite to one, were given; when the other oppofite part is found from the proportion between the fines of parts oppofite, the remaining angle, or fide, may be found by either of the two proportions foregoing.

LXXXII. *An Account of the Plants* Halefia *and* Gardenia: *In a Letter from* John Ellis, *Efq*; *F. R. S. to* Philip Carteret Webb, *Efq*; *F. R. S.*

Dear Sir,

Read Nov. 20, 1760.

YOU muft have obferved, that as the fpirit of planting has increafed in this kingdom, the ftudy of botany has become more fafhionable; the works of the celebrated Linnæus, heretofore looked on as capricious and ftrange, are now in the hands of every man, who wifhes to ftudy the order of nature.

The great variety of plants, which you have introduced into your garden from North America, as well as from many other parts of the world, muft give you double pleafure, when you view them ranged in proper order, and judicioufly named.

The